

# DYNAMICAL MODEL OF ELECTROWEAK PION PRODUCTION REACTION

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## Abstract

The dynamical model of pion electroproduction developed by the authors has been extended to study the pion weak production reaction. The axial vector  $N - \Delta$  transition form factor  $G_A^{N\Delta}$  obtained from the analysis of existing neutrino reaction data and the role of dynamical pion cloud on  $G_A^{N\Delta}$  are discussed.

## I. INTRODUCTION

An important challenge in hadron physics research is to understand the hadron structure and reaction dynamics within QCD. The electroweak form factors of baryon resonance give important informations to step forward the research in this direction. Those informations are important in testing hadron model and possibly the lattice QCD calculation. We have developed a dynamical model for investigating the pion photoproduction and electroproduction reactions in the delta resonance region [1, 2]. In the dynamical approach[1, 2, 3, 4, 5, 6] we solve the scattering equations with the interactions defined from the effective Lagrangian and/or hadron models. The dynamical approach is different from the approaches based on the dispersion relations[7-9] or K-matrix approach[10-13] in interpreting the data. In Refs. 1,2, we have not only extracted the  $N\Delta$  transition form factors from the data but also we have provided an interpretation of the extracted form factors in terms of hadron models. It was found in Refs. 1 and 2 that the pion cloud effects give large contribution to the  $N\Delta$  transition form factors. The magnetic dipole  $N\Delta$  transition form factor  $G_M^{N\Delta}(0)$  is enhanced by about 40% by meson cloud, which gives explanation on the long standing discrepancy between the prediction of the constituent quark model and empirical amplitude analyses. Furthermore, the long range pion cloud gives soft component of the  $G_M^{N\Delta}(Q^2)$  and very pronounced enhancement of quadrupole transition form factors  $G_E^{N\Delta}(Q^2), G_C^{N\Delta}(Q^2)$  at low  $Q^2$ .

The axial vector response of the hadron can be studied by the weak processes. The neutrino-induced pion production reaction  $\nu + N \rightarrow l + \pi + N$  can be used to extract such informations. The previous investigations of the weak pion production reaction[14-22] have been done by using dispersion relation approach or K-matrix approach. We report on our recent progress on the neutrino-induced pion production reaction in the delta resonance region[3]. The purpose of this work is to develop a dynamical model for neutrino-induced pion production reaction to extract axial vector  $N\Delta$  form factors by extending our model on pion electroproduction. In particular we investigate possible role of the dynamical pion cloud in solving the problem that the  $N\Delta$  axial vector form factor extracted from neutrino reaction[21, 22] was found to be about 30% larger than the quark model predictions. In section 2, we will briefly describe our dynamical model. The results of neutrino-induced pion production and the axial vector  $N\Delta$  form factor is given in section 3.

## II. DYNAMICAL APPROACH

We start from the Hamiltonian of mesons and baryons fields with the interaction Hamiltonian  $H_I = \sum_{B',M',B} \Gamma_{B'M',B}^0$ , which describes absorption and emission of meson  $M$  from baryon  $B$  as

$$H = H_0 + \Gamma_{B'M',M} + (\text{h.c.}). \quad (1)$$

Similarly the weak hadron current  $J^\mu$  consists of mesons and baryons weak currents. To obtain a manageable reaction theory to describe neutrino-induced pion production reaction, we apply unitary transformation method[1, 23] up to the second order of the interaction Hamiltonian. The idea is that we eliminate 'virtual' interaction  $B \rightarrow B'M'$  for  $m_B < m'_M + m'_B$  from the Hamiltonian in Eq. (1) and absorb their effects into many-body potentials. As a result  $N$  and  $\pi N$  states decouple with each other up to the order of our approximation and the effective Hamiltonian consists of the interactions of the resonance decay/production and the many-body potentials.

$$H_{eff} = H_0 + v_{\pi N} + \Gamma_{\Delta,\pi N} + \text{h.c.}. \quad (2)$$

Here  $v_{\pi N}$  is non-resonant  $\pi N$  potential and  $\Gamma_{\Delta,\pi N}$  describes decay of delta into  $\pi N$ . The

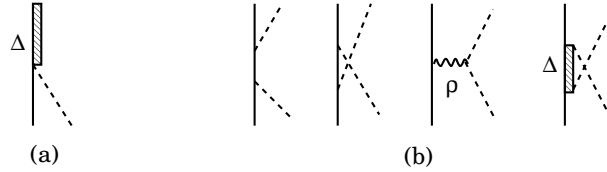


FIG. 1: Graphical representation of the interactions (a)  $\Gamma_{\Delta,\pi N}$  and (b)  $v_{\pi N}$ .

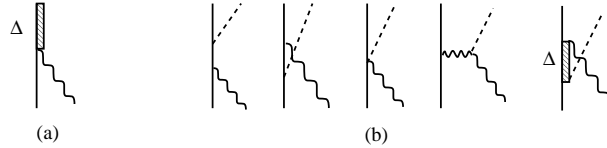


FIG. 2: Graphical representation of the effective current (a)  $J_{\Delta N}^\mu$  and (b)  $J_{\pi N}^\mu$ .

effective hadron current  $J_{eff}^\mu$  is obtained by applying the same unitary transformation used

for  $H_{eff}$  on  $J^\mu$ .  $J_{eff}^\mu$  consists of the non-resonant pion production current  $J_{\pi N}^\mu$  and  $N\Delta$  transition current  $J_{\Delta N}^\mu$ :

$$J_{eff}^\mu = J_{\Delta N}^\mu + J_{\pi N}^\mu. \quad (3)$$

From the effective Hamiltonian and current, the T-matrix of the pion production reaction can be easily obtained by solving the coupled channel Lippman-Schwinger equation within the  $\pi N \oplus \Delta$  Fock space. The resulting matrix element of the current  $\bar{J}_{\pi N}^\mu$ , which includes final state interaction and satisfies the Watson theorem, is given as

$$\bar{J}_{\pi N}^\mu = \bar{J}_{\pi N}^\mu(\text{non-res}) + \frac{\bar{\Gamma}_{\Delta, \pi N} \bar{J}_{\Delta N}^\mu}{W - m_\Delta - \Sigma}. \quad (4)$$

The non-resonant current  $\bar{J}_{\pi N}^\mu(\text{non-res})$  is calculated only from non-resonant interactions  $v_{\pi N}$  and  $J_{\pi N}^\mu$ . The second term in Eq. (4) is resonant amplitude with the dressed  $J^\mu + N \rightarrow \Delta$  vertex given as

$$\bar{J}_{\Delta N}^\mu(W, q) = J_{\Delta N}^\mu + \int dk k^2 \frac{\bar{\Gamma}_{\Delta, \pi N}(W, k) J_{\pi N}^\mu(k, q)}{W - E_N(k) - E_\pi(k) + i\epsilon}. \quad (5)$$

An important feature of the dynamical model is that the bare vertex  $J_{\Delta N}^\mu$ , which may be compared with the prediction of the hadron model, is modified by the off-shell non-resonant interaction  $J_{\pi N}^\mu$  to give the dressed vertex  $\bar{J}_{\Delta N}^\mu$ .

### III. WEAK PION PRODUCTION REACTION

We apply the method described in the previous section to  $\nu_\mu + N \rightarrow \mu + \pi + N$  reaction. The low energy effective Lagrangian of the electroweak standard model is given as

$$H_W = \frac{G_F \cos_c}{\sqrt{2}} (V^\mu - A^\mu) l_\mu. \quad (6)$$

Here  $V^\mu$  and  $A^\mu$  are charged currents of hadron and  $l^\mu$  is lepton current. The vector current ( $V^\mu$ ) is obtained from the electromagnetic current in Refs. 1,2 by assuming CVC and iso-spin rotation. The axial vector hadron current is obtained from the chiral Lagrangian. Following the procedure described in the previous section, the pion production currents  $J_{eff}^\mu$  are calculated shown in the schematic diagrams of Fig. 2. We took the constituent quark model relation of the coupling constant of the axial  $N\Delta$  current  $A_{\Delta N}^\mu$  is obtained from the

quark model relation  $G_A^* = \sqrt{72/25}g_A$ . The  $Q^2$  dependence of the axial form factor is assumed as

$$G_A^*(Q^2) = G_A(0) \frac{1}{(1 + Q^2/m_A^2)^2} R_{SL}(Q^2), \quad (7)$$

where the dipole form factor is axial vector form factor of nucleon with  $m_A = 1.02\text{GeV}$  [24] and phenomenological form factor  $R_{SL}(Q^2) = (1 + aQ^2)\exp(-bQ^2)$  with  $a = 0.154\text{GeV}^{-2}$  and  $b = 1.66\text{GeV}^{-2}$  is obtained by fitting pion electroproduction reaction cross section at  $Q^2 = 2.8$  and  $4(\text{GeV}/c)^2$ . All the other coupling constants and the form factors used in this work are the same value as our model of the pion electroproduction. Therefore, there is no adjustable parameters in this calculation.

We first compare the total cross section of  $\nu_\mu + p \rightarrow \mu^- + \pi^+ + p$  reaction with the data [26] in Fig. 3. Our result shown in solid curve agrees reasonably well with the data. One of the main feature of the dynamical approach is the bare form factor of  $N\Delta$  transition is renormalized by the pion rescattering effects. The importance of this effect is seen in Fig. 3. The full result in solid line is reduced into dashed line when we turn off the dynamical pion cloud effect. The contribution of the pure non-resonant contribution is shown in dot-dashed line. We next compare the  $Q^2$  dependence of the calculated differential cross section with the data [25] in Fig. 4. Our model reproduce very well the data. In the low  $Q^2$  region, the main contribution is the the axial vector current shown in the dashed line. The BNL data were used in the most recent attempt to extract  $N\Delta$  axial vector form factor.

Finally we study the effect of dynamical pion cloud effect on the axial vector form factor. The empirical  $N\Delta$  form factor extracted from the data can be compared with our dressed form factor  $\bar{J}_{\Delta N}^\mu$  shown in the solid line in Fig. 5, while the contribution of the bare form factor  $J_{\Delta N}^\mu$  is shown in the dotted line. We see sizable contribution of the dynamical pion cloud as the case of the electromagnetic form factors. The differences between solid and dotted lines explain the observation that the quark model prediction of axial  $N\Delta$  form factor is about 30% smaller than the empirically extracted form factor.

#### IV. SUMMARY

In summary, the dynamical model developed in [1, 2, 3] can describe extensive data of pion photoproduction, electroproduction and the neutrino-induced reactions. The predicted

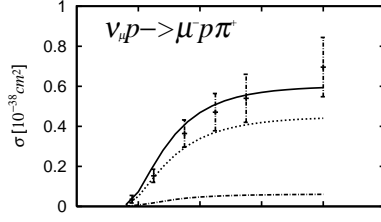


FIG. 3: Total cross section of  $p + \nu_\mu \rightarrow \mu^- + \pi^+ + p$  reaction.

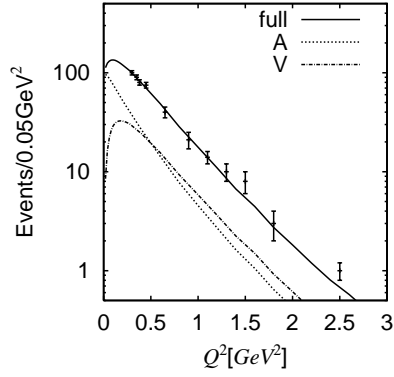


FIG. 4: Differential cross section  $d\sigma/dQ^2$  of  $p + \nu_\mu \rightarrow \mu^- + \pi^+ + p$  reaction

$\nu_\mu + N \rightarrow \pi + N + \mu$  cross sections are in good agreement with the existing data. The renormalized axial vector  $N\Delta$  form factor contains large dynamical pion cloud effects and these effects are crucial in getting agreement with the data. We conclude that the  $N\Delta$  transition axial vector form factor predicted from the quark model is consistent with the

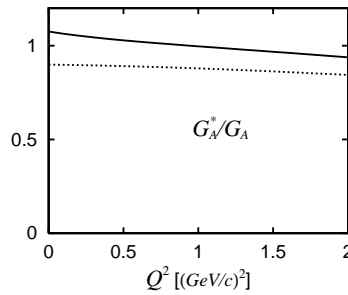


FIG. 5: The  $N\Delta$  axial vector form factor.

existing data in the delta region. However more extensive and precise data of neutrino-induced pion production reactions are needed to further test our model and to pin down the  $Q^2$  dependence of the form factors. The efforts to extend the current dynamical model beyond the delta resonance region[27] and to improve the effective Hamiltonian including leading loop corrections[28] are in progress.

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